QP Code : 541304

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. **one** is **compulsory**.
(2) Answer **any three** questions from **Q.2 to Q.6**
(3) Use of stastical Tables permitted.
(4) Figures to the **right** indicate **full** marks
(5) Assume suitable data wherever applicable.

1. (a) Find the Eigenvalues and eigenvectors of the matrix. 5

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) Evaluate the line integral $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x$ 5

(c) Find k and then E (x) for the p.d.f. 5

$$f(x) = \begin{cases} k(x-x^2) & ,0 \le x \le 1, k > 0 \\ 0 & , \text{ otherwise} \end{cases}$$

(d) Calculate Karl pearson's coefficient of correlation from the following data. 5

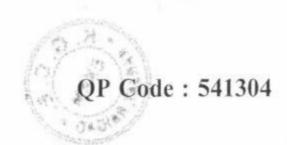| x	100	200	300	400	500
y	30	40	50	60	70

2. (a) Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non- derogatory. 6

(b) Evaluate $\int \dfrac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z-1|=3$ 6

(c) If x is a normal variate with mean 10 and standard deviation 4 find 8
(i) $P(|x-14|<1)$ (ii) $P (5 \le x \le 18)$ (ii) $P (x \le 12)$

3. (a) Find the relative maximum or minimum (if any) of the function   **6**
$Z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$

  (b) If x is Binomial distributed with $E(x) = 2$ and $V(x) = 4/3$, find the   **6**
probability distribution of x.

  (c) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A^{50}.   **8**

4. (a) Solve the following L.P.P. by simplex method   **6**

Minimize $z = 3x_1 + 2x_2$
Subject to $3x_1 + 2x_2 \le 18$
               $0 \le x_1 \le 4$
               $0 \le x_2 \le 6$
               $x_1, x_2 \ge 0.$

  (b) The average of marks scored by 32 boys is 72 with standard deviation 8   **6**
while that of 36 girls is 70 with standard deviation 6. Test at 1% level of
significance whether the boys perform better than the girls.

  (c) Find Laurent's series which represents the function $f(z) = \dfrac{2}{(z-1)(z-2)}$ **8**

When (i) $|z| < 1$,   (ii) $1 < |z| < 2$ (iii) $|z| > 2$

5. (a) Evaluate $\displaystyle\int_c \dfrac{z^2}{(z-1)^2(z+1)}\, dz$ where C is $|z| = 2$ using residue theorem   **6**

  (b) The regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$ Find   **6**

    (i)   Sample means $\bar{x}$ and $\bar{y}$

    (ii)   Correlation coefficient between x and y. Also estimate y When
          $x = 12$

  (c) A die was thrown 132 times and the following frequencies were observed   **8**

No.obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Using χ^2-test examine the hypothesis that the die is unbiased.

6. (a) Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2 + x + 2}{x^4 + 10x^2 + 9} \, dx$ using contour integration. **6**

(b) If a random variable x follows Poisson distribution such that $P(x = 1) = 2 P(x = 2)$ Find the mean and the variance of the distribution. Also find $P(x=3)$. **6**

(c) Use Penalty method to solve the following L.P.P. **8**
Minimize $\quad z = 2x_1 + 3x_2$
Subject to $\quad x_1 + x_2 \geq 5$
$\qquad\qquad x_1 + 2x_2 \geq 6$
$\qquad\qquad x_1 , x_2 \geq 0.$